A Short Introduction to BIT-STRING PHYSICS * †

H. Pierre Noyes
Stanford Linear Accelerator Center
Stanford University, Stanford, CA 94309

Abstract

This paper starts with a personal memoir of how some significant ideas arose and events took place during the period from 1972, when I first encountered Ted Bastin, to 1979, when I proposed the foundation of ANPA. I then discuss program universe, the fine structure paper and its rejection, the quantitative results up to ANPA 17 and take a new look at the handy-dandy formula. Following this historical material is a first pass at establishing new foundations for bit-string physics. An abstract model for a laboratory notebook and an historical record are developed, culminating in the bit-string representation. I set up a tic-toc laboratory with two synchronized clocks and show how this can be used to analyze arbitrary incoming data. This allows me to discuss (briefly) finite and discrete Lorentz transformations, commutation relations, and scattering theory. Earlier work on conservation laws in 3- and 4- events and the free space Dirac and Maxwell equations is cited. The paper concludes with a discussion of the quantum gravity problem from our point of view and speculations about how a bit-string theory of strong, electromagnetic, weak and gravitational unification could take shape.

Revised and considerably extended version of two invited lectures presented at the 18^{th} annual international meeting of the

ALTRNATIVE NATURAL PHILOSOPHY ASSOCIATION

Wesley House, Cambridge, England, September 4-7, 1996

^{*}Work supported by Department of Energy contract DE-AC03-76SF00515.

[†]Conference Proceedings, entitled *Merologies*, will be available from ANPA c/o Prof.C.W.Kilmister, Red Tiles Cottage, Hight Street, Bascombe, Lewes, BN8 5DH, United Kingdom.

1 Pre-ANPA IDEAS: A personal memoir

1.1 First Encounters

When I first met Ted Bastin in 1972 and heard of the Combinatorial Hierarchy (hereinafter CH), my immediate reaction was that it must be dangerous nonsense. Nonsense, because the two numbers computed to reasonable accuracy — $137 \approx \hbar c/e^2$ and $2^{127} + 136 \approx \hbar c/Gm_p^2$ — are empirically determined, according to conventional wisdom. Dangerous, because the idea that one can gain insight into the physical world by "pure thought" without empirical input struck me then (and still strikes me) as subversive of the fundamental Enlightenment rationality which was so hard won, and which is proving to be all too fragile in the "new age" environment that the approach to the end of the millennium seems to encourage [84, 86].

Consequently when Ted came back to Stanford the next year (1973)[8], I made sure to be at his seminar so as to raise the point about empirical input with as much force as I could. Despite my bias, I was struck from the start of his talk by his obvious sanity, and by a remark he made early on (but has since forgotten) to the effect that the basic quantization is the quantization of mass. When his presentation came around to the two "empirical" numbers, I was struck by the thought that some time ago Dyson[19] had proved that if one calculates perturbative QED up to the approximation in which 137 electron-positron pairs can be present, the perturbation series in powers of $\alpha = e^2/\hbar c \approx 1/137$ is no longer uniformly convergent. Hence, the number 137 as a counting number already had a respectable place in the paradigm for relativistic quantum field theory known as renormalized quantum electrodynamics (QED). The problem for me became why should the arguments leading to CH produce a number which also supports this particular physical interpretation.

As to the CH itself, I refer you to Clive Kilmister's introductory talk in these proceedings[39], where he discusses an early version of the bit-string construction of the sequence of discriminately closed subsets with cardinals $2^2 - 1 = 3 \rightarrow 2^3 - 1 = 7 \rightarrow 2^7 - 1 = 127 \rightarrow 2^{127} - 1 \approx 1.7 \times 10^{38}$ based on bit-strings of length 2,4,16,256 respectively. The first three terms can be mapped by square matrices of dimension $2^2 = 4 \rightarrow 4^2 = 16 \rightarrow 16^2 = 256$. The 256² discriminately independent matrices made available by squaring the dimension needed to map the third level are many two few to map the $2^{127}-1$ discriminately closed subsets in the fourth level, terminating the construction. In the historical spirit of this memoir, I add that thanks to some archeological work John Amson and I did in John's attic in St. Andrews, the original paper on the hierarchy by Fredrick Parker-Rhodes, drafted late in 1961, is now available[79].

I now ask you to join with me here in my continuing investigation of how the CH can be connected

to conventional physics. As you will see in due course, this research objective differs considerably from the aims of Ted Bastin and Clive Kilmister. They, in my view, are unnecessarily dismissive of the results obtained in particle physics and physical cosmology using the conventional (if mathematically inconsistent) relativistic quantum field theory, in particular quantum electrodynamics (QED), quantum chromodynamics (QCD) and weak-electromagnetic unification (WEU).

Before we embark on that journey, I think it useful to understand some of the physics background. Dyson's argument itself rests on one of the most profound and important papers in twentieth century physics. In 1937 Carl Anderson discovered in the cosmic radiation a charged particle he could show to be intermediate in mass between the proton and electron. This was the first to be discovered of the host of particles now called collectively "mesons". One such particle had already been postulated by Yukawa, a sort of "heavy photon" which he showed, using a "massive QED", gave rise to an exponentially bounded force of finite range. If the mass of the Yukawa particle was taken to be a few hundred electron masses, this could be the "nuclear force quantum". Anderson's discovery prompted Gian Carlo Wick to try to see if the existence of such a particle could be accounted for simply by invoking the basic principles of quantum mechanics and special relativity. He succeeded brilliantly, using only one column in Nature[96]. We summarize his argument here.

Consider two massive particles which are within a distance R of each other during a time Δt . If they are to act coherently, we must require $R \leq c\Delta t$. [Note that this postulate, in the context of my neo-operationalist approach[68] based on measurement accuracy[76], opens the door to supraluminal effects at short distance, which I am now starting to explore[69]]. Because of the uncertainty principle this short-range coherence tells us that the energy is uncertain by an amount $\Delta E \approx \hbar/\Delta t$. But then mass-energy equivalence allows a particle of mass μ or rest-energy $\mu c^2 \geq \Delta E$ to be present in the space time-volume of linear dimension $\approx R\Delta t$. Putting this together, we have the Wick-Yukawa Principle:

$$R \le c\Delta t \approx \frac{c\hbar}{\Delta E} \le \hbar/\mu c \tag{1}$$

Put succinctly, if we try to localize two massive particles within a distance $R \leq \hbar/\mu c$, then the uncertainty principle allows a particle of mass μ to be present. If this meson has the proper quantum numbers to allow it to transfer momentum between the two massive particles we brought together in this region, they will experience a force, and will emerge moving in different directions than those with which they entered the *scattering region*. Using estimates of the range of nuclear forces obtained from deviations from Rutherford scattering in the 1930's one can then estimate the mass of the "Yukawa particle" to be $\approx 200 - 300$ electron masses.

We are now ready to try to follow Dyson's argument. By 1952, one was used to picturing the result of Wick-Yukawa uncertainty at short distance as due to "vacuum fluctuations" which would

allow N_e electron-positron pairs to be present at distances $r \leq \hbar/2Nm_ec$. This corresponds to taking $\mu = 2Nm_e$ in Eq. 1. Although you will not find it in the reference[19], in a seminar Dyson gave on this paper he presented what he called a crude way to understand his calculation making use of the non-relativistic coulomb potential. I construct here my own version of the argument.

Consider the case where there are N_e positive charges in one clump and N_e negative charges in the other, the two clumps being a distance $r = \hbar/m_e c$ apart. Then a single charge from one clump will have an electrostatic energy $N_e e^2/r = N_e [e^2/\hbar c] m_e c^2$ due to the other clump and visa versa. I do recall that Dyson said that the system is dilute enough so that non-relativistic electrostatic estimates of this type are a reasonable approximation. If under the force of this attraction, these two charges we are considering come together and scatter producing a Dalitz pair $(e^+ + e^- \rightarrow 2e^+ + 2e^-)$ the energy from the fluctuation will add another pair to the system. Of course this process doesn't happen physically because like charges repel and the clumps never form in this way. However, in a theory in which like charges attract [which is equivalent to renormalized QED with $\alpha_e = [e^2/\hbar c] \rightarrow -\alpha_e$ in the renormalized perturbation series], once one goes beyond 137 terms such a process will result in the system gaining energy by producing another pair and the system collapses to negatively infinite energy. Dyson concluded that the renormalized perturbation series cannot be uniformly convergent, and hence that QED cannot be a fundamental theory, as I have subsequently learned from Schweber's history of those heroic years[85].

Returning to 1973, once I had understood that, thanks to Dyson's argument, 137 can be interpreted as a counting number, I saw immediately that $2^{127} + 136 \approx 1.7 \times 10^{38} \approx \hbar c/Gm_p^2$ could also be interpreted as a counting number, namely the number of baryons of protonic mass which, if found within the Compton wavelength of any one of them, would form a black hole. These two observations removed my objection to the calculation of two pure numbers that, conventionally interpreted, depend on laboratory measurements using arbitrary units of mass, length and time. I could hardly restrain my enthusiasm long enough to allow Ted to finish his seminar before bursting out with this insight. If this cusp turns out to be the point at which a new fundamental theory takes off — as I had hoped to make plausible at ANPA 18 — then we can tie it firmly into the history of "normal science" as the point where a "paradigm shift", in Kuhn's sense of the word[40], became possible.

However, my problem with why the calculation made by Fredrick Parker-Rhodes[79, 80] lead to these numbers remained unresolved. Indeed, I do not find a satisfactory answer to that question even in Ted and Clive's book published last year[10]. [I had hoped to get further with that quest at the meeting (ANPA 18, Sept.5-8, 1996), but discussions during and subsequent to the meeting still leave many of my questions unanswered. I intend to review these discussions and draw my own

1.2 From "NON-LOCALITY" to "PITCH": 1974-1979

My interest in how to resolve this puzzle has obviously continued to this day. I was already impressed by the quality of Fredrick's results in 1973, and made a point of keeping in contact with Ted Bastin. This led to my meeting with Fredrick Parker-Rhodes, Clive Kilmister and several of the other Epiphany Philosophers at a retreat in the windmill at Kings Lynn, followed by discussions in Cambridge. At that point the group were trying to put together a volume on *Revisionary Philosophy and Science*. I agreed to contribute a chapter, and finished about half of the first draft of what was to become "Non-Locality in Particle Physics" [55, 56] on the plane going back to Stanford. In the course of finishing that article, I noted for the first time that the Dyson route to the hierarchy number places an energy cutoff on the validity of QED at $E_{max} = 2 \times 137 m_e c^2$, which is approximately equal to the pion (Yukawa particle) mass. I have subsequently realized that this explains a puzzle I had been carrying with me since I was a graduate student.

This puzzle, which I have sometimes called the *Joe Weinberg memnonic*, came from quite another direction[91]. An easy way to remember the hierarchy of nuclear, QED, and atomic dimensions expressed in terms of fundamental constants is the fact that

$$1.4 \ Fermi \approx \frac{e^2}{2m_e c^2} = \left[\frac{e^2}{\hbar c}\right] \frac{\hbar}{2m_e c} = \left[\frac{e^2}{\hbar c}\right]^2 \frac{\hbar^2}{2m_e e^2} \approx 0.265 \ Angstrom \tag{2}$$

Why nuclear dimensions should be approximately half the "classical electron radius" (i.e. $\frac{e^2}{2m_ec^2} \approx 1.4 \times 10^{-15} meter$) and hence $[1/137]^2$ smaller than than the radius of the positronium atom (i.e. $\frac{\hbar^2}{2m_ee^2} \approx 2.65 \times 10^{-10} meter$) was almost completely mysterious in 1947. It was known that the mass of the electron attributed to it's electrostatic "self-energy" as due to its charge distributed over a spherical shell fixed at this radius would have the mass m_e , but the success of Einstein's relativity had shown that this electron model made no sense[78]. The square of this parameter was also known to be proportional to the cross section for scattering a low energy electromagnetic wave from this model electron (Thompson cross section $[8\pi/3](e^2/m_ec^2)^2$), but again why this should have anything to do with nuclear forces was completely mysterious.

As we have already seen, it was known that the Wick-Yukawa principle [96] accounted roughly for the range of nuclear forces if those forces were attributed to a strongly interacting particle intermediate in mass between proton and electron. However, the only known particle in that mass range (the muon) had been shown experimentally to interact with nuclei with an energy 10¹³ times smaller than the Yukawa theory of nuclear forces demanded[18]. The Yukawa particle (the pion)

was indeed discovered later that year, but there was still no reason to connect it with the "classical electron radius". Joseph Weinberg left his students to ponder this puzzle.

The trail to the solution of this conundrum starts with a 1952 paper by Dyson[19], despite the fact that neither he nor I realized it at the time. Two decades later, when I first heard a detailed account of the combinatorial hierarchy[8], and was puzzled by the problem of how a counting number (i.e. 137) could approximate a combination of empirical constants (i.e. $\hbar c/e^2$), I realized that this number is both the number of terms in the perturbation series and the number of virtual electron-positron pairs where QED ceases to be self-contained. But, empirically, $m_{\pi} \approx 2 \times 137 m_e$. Of course, if neutral this system is highly unstable due to 2γ decay, but if we add an electron-antineutrino or a positron-neutrino pair to the system, and identify the system with π^- or π^+ respectively, the system is stable until we include weak decays in the model. This suggests that the QED theory of electrons, positrons and γ -rays breaks down at an energy of $[2(\hbar c/e^2) + 1]m_ec^2$ due to the formation of charged pions, finally providing me with a tentative explanation for the Joe Weinberg memnonic. As noted above, I first presented this speculative idea some time ago [55, 56].

By the time I wrote "NON-LOCALITY", I was obviously committed to engaging in serious research on the combinatorial hierarchy as part of my professional activity. Ted was able to get a research contract to spend a month with me at Stanford. I had hoped that this extended period of interaction would give me a better understanding of what was going on; in the event little progress was made on my side. By 1978 I had met Irving Stein, and was also struggling to understand how he could get both special relativity and the quantum mechanical uncertainty principle from an elementary random walk. His work, after much subsequent development, is now available in final form [88].

Meanwhile Ted had attended the 1976 Tutzing Conference organized by Carl Friedrick von Weizsacker and presented a paper on the combinatorial hierarchy by John Amson. I agreed to accompany Ted to the 1978 meeting and present a joint paper. I arrived in England to learn of the startlingly successful successful calculation of the proton-electron mass ratio, which Ted and I had to discuss and digest in order to present Fredrick's result [11, 81] at the Tutzing meeting, which followed almost immediately thereafter. This formula has been extensively discussed at ANPA meetings. It was originally arrived at by assuming that the electron's charge could come apart, as a statistical fluctuation, in three steps with three degrees of freedom corresponding to the three dimensions of space and that the electrostatic energy corresponding to these pieces could be computed by taking the appropriate statistical average cut off at the proton Compton radius \hbar/m_pc . The only additional physical input is the CH value for the electronic charge $e^2 = \hbar c/137$. Take $0 \le x \le 1$ to be the fractional charge in these units and x(1-x) the charge factor in Coulomb's

law. Take $0 \le y \le 1$ to be the inverse distance between the charge fractions in that law in units of the proton Compton radius. Then, averaging between these limits with the appropriate weighting factors of $x^2(1-x)^2$ and $1/y^3$ respectively, Fredrick's straightforward statistical calculation gives

$$\frac{m_p}{m_e} = \frac{137\pi}{\langle x(1-x) \rangle \langle \frac{1}{y} \rangle} = \frac{137\pi}{(\frac{3}{14})[1+\frac{2}{7}+\frac{4}{49}](\frac{4}{5})}$$
(3)

At that time the result was within a tenth of a standard deviation of the accepted value. I knew this was much too good because, for example, the calculation does not include the effect of the weak interactions. I was therefore greatly relieved when a revision of the fit to the fundamental constants changed the empirical value by 20 standard deviations, giving us something to aim at when we know how to include additional effects.

I also learned from the group during those few days before Tutzing that up to that point no one had proved the *existence* of the combinatorial hierarchy in a mathematical sense! Subsequent to the Tutzing meeting, thanks to the kind hospitality of K.V.Laurikainen in Finland, I was able to devote considerable time to an empirical attack on that problem and get a start on actually *constructing* specific representations of both the *level* $2 \longrightarrow level$ 3 and the *level* $3 \longrightarrow level$ 4 mappings.

It turned out that neither John Amson's nor our contributions to the Tutzing conferences, despite promises, appeared in the conference proceedings. Fortunately we had had an inkling at the meeting that this contingency might arise. In the event we were able to turn to David Finkelstein and write a more careful presentation of the developments up to that point for publication in the *International Journal of Theoretical Physics*[11]. The first version, called "Physical Interpretation of the Combinatorial Hierarchy" (or PICH for short) still lacked a formal existence proof, but Clive came up with one; further, he and John Amson (whose unpublished 1976 Tutzing contribution had been extended and completed to serve as an Appendix) were able to say precisely in what sense the CH is *unique*. The final title was therefore changed to "Physical Interpretation and mathematical structure of The Combinatorial Hierarchy" affectionately known as PITCH. The finishing touches on this paper were completed at the first meeting of ANPA. This brings my informal history to the point at which Clive ended his historical sketch in his first lecture.

1.3 ANPA 1: The foundation of the organization

Although I was obviously putting considerable time into trying to understand the CH, and the Parker-Rhodes formula for m_p/m_e showed that there might be more to the physics behind it than the basic coupling constants, I was by no means convinced that the whole effort might not turn out in the long run to be an unjustifiable "numerology". I therefore, privately, took the attitude that my efforts should go into trying to derive a clear contradiction with empirical results which

would prove the CH approach to be wrong. Then I could drop the whole enterprise and get back to my (continuing) conventional research, where I felt more at home. I was not the only one with doubts at this time. Clive told us, years later, that he had been somewhat afraid to examine the foundational arguments too closely for fear that the whole scheme would dissolve!

In the spring of 1979 I happened to make the acquaintance of an investment counselor named Dugal Thomas who was advising a large fraction of the private charitable foundations in the US. He offered to help me with fundraising if I could put together a viable organization for supporting Ted Bastin's type of research. I threw together a proposal very quickly. Dugal located a few prospective donors; like all subsequent efforts to raise substantial funds for ANPA this initial effort came a cropper. Soon after that effort started I also learned that I had received a Humboldt U.S.Senior Scientist award, giving me the prospect of a year in Germany and some extra cash. Consequently I felt encouraged to approach Clive to see if he would serve as treasurer for the proposed organization. Clive agreed to approach Fredrick to see if he would match the small amount of "seed money" I was prepared to invest in ANPA. [The name and original statement of purpose came from the proposal I had already written. I intended that the term "natural philosophy" in the name of the organization would hark back to the thinkers at the start of the scientific revolution who were trying to look at nature afresh and shake themselves loose from the endless debates of the "nominalist" and "realist" metaphysicists of the schools.] With Fredrick's promise in hand, Clive and I approached Ted Bastin with the invitation to be the Coordinator, and asked John Amson to join us as a founding member.

The result of all this was what can be properly called ANPA 1, which met in Clive's Red Tiles Cottage near Lewes in Sussex in the early fall of 1979. John Amson was unable to attend, but endorsed our statement of purpose (modified by Ted to include specific mention of the CH) and table of organization. Once these details were in hand we had a proper scientific meeting, including thrashing out an agreed manuscript for PITCH. I gave a paper on the quantum mechanical three and four body problem, which I was working on in Germany. I noted in particular that the three channel Faddeev equations go to the seven channel Faddeev-Yakubovsky equations when one goes from three to four independent particles, reminiscent of the CH $1 \longrightarrow 2$ level transition. It is taken a long time to see what the relationship is between these two facts, but now that I am developing a "bit-string scattering theory" with Ed Jones[75], this old insight is finding an appropriate home.

Selected Topics

All meetings subsequent to ANPA 1 have been held annually in Cambridge, England. Proceedings were prepared for ANPA 7[58], and some of the papers incorporated in a SLAC-PUB[59]. The ANPA 9 proceedings [60] are available from ANPA West. Proceedings ANPA's 10 to 17 are

available from Clive Kilmister. This is obviously not the place to attempt the impossible task of summarizing 16 years of work by more than 20 dedicated people in a way that would do justice to their varied contributions. I have therefore chosen to pick a few topics where I still find continued discussion both interesting and important.

2 Program Universe

2.1 Origin of Program Universe

About a decade and a half ago, Clive attempted to improve the clarity of what Ted has called "the canonical approach" [9] by admitting into the scheme a second operation besides the *Discrimination* operation, which had been central to the project ever since John Amson introduced it [2] and related it to subsequent developments [4]. Clive called this second operation *Generation* because at that stage in his thinking he saw no way to get the construction off the ground without generating bit-strings as well as discriminating them. I think he had in mind at that time a random sequence of G and D operations, but did not quite know how to articulate it. Because Mike Manthey and I were unsure how to construct a specific theory from what we could understand of this new approach, we decided to make a simple-minded computer model of the process and see how far it would lead. The first version [42] turned out to be unnecessarily complicated, and was replaced [73] by the version described below in section 2.3.

One essential respect in which the construction Mike Manthey and I turned out differs from the canonical approach is that we explicitly introduced a random element into the generation of the bit-strings rather than leaving the background from which they arise vague. Some physicists, in particular Pauli, have seen in the random element that so far as proved to be inescapable in the discussion of quantum phenomena an entrance of the the "irrational" into physics. This seems to me to equate "rationality" with determinism. I think this is too narrow a view. Statistical theories of all sorts are used in many fields besides physics without such approaches having to suffer from being castigated as irrational. In particular, biology is now founded on the proposition that evolution is (mainly) explicable as the natural selection of heritable stability in the presence of a random background. The caveat "mainly" is inserted to allow for historical contingencies[33]. Even in physics, the idea of a random "least step" goes back at least to Epicurus, and of a least step to Aristotle. I would characterize Epicurus as an exemplary rationalist whose aim was to help mankind escape from the superstitious terrors generated by ancient religions. This random element enters program universe via the primitive function "flipbit" which Manthey uses to provide either a zero or a one by unsynchronized access to a closed circuit that flips these two bits back and forth

between two memory locations. Before discussing how this routine is used, we need to know a bit more about bit-strings and the operations by which we combine them.

2.2 Bit-Strings

Define a bit-string $\mathbf{a}(\mathbf{a}; \mathbf{W})$ with length W and Hamming measure a by its W ordered elements $(\mathbf{a})_w \equiv a_w \in 0, 1$ where $w \in 1, 2, ..., W$. Define the Dirac inner product, which reduces two bit-strings to a single positive integer, by $\mathbf{a} \cdot \mathbf{b} \equiv \sum_{w=1}^W a_w b_w$. Hence $\mathbf{a} \cdot \mathbf{a} = a$ and $\mathbf{b} \cdot \mathbf{b} = b$. Define discrimination between bit-strings of the same length, which yields a third string of the same length, by $(\mathbf{a} \oplus \mathbf{b})_w = (a_w - b_w)^2$. Clive and I arrived at this way of representing discrimination during a session in his office after ANPA 2 or 3. From this representation the basic bit-string theorem follows immediately:

$$(\mathbf{a} \oplus \mathbf{b}) \cdot (\mathbf{a} \oplus \mathbf{b}) = a + b - 2\mathbf{a} \cdot \mathbf{b} \tag{4}$$

This equation could provide the starting point for an alternative definition of " \oplus " which avoids invoking the explicit structure used above.

We also will need the *null string* $\Phi(W)$ which is simply a string of W zeros. Note that $\mathbf{a} \oplus \mathbf{a} = \Phi(W)$, that $(\mathbf{a} \oplus \mathbf{a}) \cdot (\mathbf{a} \oplus \mathbf{a}) = 0$ and that $\mathbf{a} \cdot \Phi = 0$. The complement of the null string is the anti-null string $\mathbf{W}(W)$ which consists of W ones and has the property $\mathbf{W} \cdot \mathbf{W} = W$. Of course $\mathbf{W} \cdot \Phi = 0$.

Define concatenation, symbolized by " $\|$ ", for two string $\mathbf{a}(a; S_a)$ and $\mathbf{b}(b; S_a)$ with Hamming measures a and b and respective lengths S_a and S_b and which produces a string of length $S_a + S_b$, by

$$(\mathbf{a}\|\mathbf{b})_{s} \equiv a_{s} \ if \ s \in 1, 2, ..., S_{a}$$

$$\equiv b_{S_{a}-s} \ if \ s \in S_{a}+1, S_{a}+2, ..., S_{a}+S_{b}$$
(5)

For strings of equal length this doubles the length of the string and hence doubles the size of the bit-string space we are using. For strings of equal length it is sometimes useful to use the shorthand but somewhat ambiguous "product notation" **ab** for concatenation. Note that while "·" and " \oplus " are, separately, both associative and commutative, in general concatenation is not commutative even for strings of equal length, although it is always, separately, associative.

2.3 Program Universe

To generate a growing universe of bit-strings which at each step contains P(S) strings of length S, we use an algorithm known as program universe which was developed in collaboration with

M.J.Manthey [42, 73]. Since no one knows how to construct a "perfect" random number generator, we cannot start from Manthey's "flipbit", and must content ourselves with a pseudo-random number generator that, to some approximation which we will be wise to reconsider from time to time, will give us either a "0" or a "1" with equal probability. Using any available approximation to "flipbit" and assigning an order parameter $i \in 1, 2, ..., P(S)$ to each string in our array, Manthey[42] has given the coding for constructing a routine "PICK" which picks out some arbitrary string $\mathbf{P}_i(S)$ with probability 1/P(S). Then program universe amounts to the following simple algorithm:

PICK any two strings $\mathbf{P}_i(S)$, $\mathbf{P}_j(S)$, $i, j \in 1, 2, ..., P$ and compare $\mathbf{P}_{ij} = \mathbf{P_i} \oplus \mathbf{P_j}$ with $\mathbf{\Phi}(S)$.

If $\mathbf{P}_{ij} \neq \mathbf{\Phi}$, adjoin $\mathbf{P}_{P+1} := \mathbf{P}_{ij}$ to the universe, set P := P+1 and recurse to PICK. [This process is referred to as ADJOIN.]

Else, for each $i \in 1, 2, ..., P$ pick an arbitrary bit $\mathbf{a}_i \in 0, 1$, replace $\mathbf{P}_i(S+1) := \mathbf{P}_i(S) \| \mathbf{a}_i$, set S := S+1 and recurse to PICK. [This process is referred to as TICK.]

It is important to realize that if we take a snapshot of the universe of bit-strings so constructed at any time, with the P_i written as rows of 0's and 1's in a rectangular array containing S columns, there is nothing in the process that generated them which distinguishes this universe from any of the S! other universes of 0's and 1's of this height and width which could be obtained by using any of the S! possible permutations of the columns. In this sense any run of program universe up to this point could just as well have produced any of these other universes. The point here is that, since the rows are produced by discrimination, and the order of the bits is the same for each row, the result is independent of the order of the bits. Similarly, since the column of bits which is adjoined to this block representation just before $S \to S+1$ is some (hopefully good!) approximation to a Bernoulli sequence, the probability of it having k 1's and P(S) - k 0's is simply P(S)!/k!(P(S)-k)! independent of how the rows are ordered by the order parameter i. That is, even though we have introduced an order parameter for the rows in order to make it easy to code the program in a transparent way, this parameter in itself is not intended to play any role in the physical interpretation of the model. At this stage in our argument, this means that program universe can end up with any one of the $2^{P(S)}S!$ possible block rectangles containing only 0's and 1's of height P(S) and width S with some probability which is presumably calculable. This probability is relevant when we come to discuss cosmology. Nevertheless, if we look at the *internal structure* of some fixed portion of any one of these universes, the way in which they are constructed will allow us to make some useful and general statements. Further, these rectangular blocks of "0" 's and "i" 's are tables and hence have shapes in the precise sense defined by Etter's Link Theory [22, 23, 24, 25, 26]. I hope to have time to discuss Etter's theory at ANPA 19.

I have called another symmetry of the universes so constructed *Amson invariance* in reference to his paper on the BI-OROBOUROS [5]. He notes that there is nothing in the discrimination operation which prevents us from using the alternative representation for discrimination given by

$$0 \oplus' 0 = 1; \ 0 \oplus' 1 = 1 = 1 \oplus' 0; \ 1 \oplus' 1 = 1$$
 (6)

This will produce a dual representation of the system in which the roles of the bits "0" and "1" (which obviously can no longer be thought of as integers in a normal notation) are interchanged. Then when the construction of the combinatorial hierarchy is completed at level 4, one will have the complete system and its dual. But then, one can answer the question which has been asked in these meetings: "Where do the bits in the CH come from?" in an interesting way. In John's construction the bits are simply the two dual representations of the CH! Consequently one has a nested sequence of CH's with no beginning and no end. The essential point for me here is not this nested sequence—which will be difficult to put to empirical test—but the emphasis it gives to the fact that the two symbols are arbitrary and hence that their interchange is a symmetry operation. This has helped me considerably in thinking about how the particle-antiparticle symmetry and CPT invariance come about in bit-string physics.

Note that in the version of program universe presented here the arbitrary bits are concatenated only at one growing end of the strings. Consequently, once the string length S passes any fixed length L the P(L) strings present will consist of some number $n_L \leq L$ of strings which are discriminately independent. Further, once S > L, the portion of all string of length L changes only by discrimination between members of this collection. Consequently it can end up containing at most $2^{n_L} - 1$ types of distinct, non-null strings no matter how much longer program universe runs. Whether it ever even reaches this bound, and the value of n_L itself, are historically contingent on which run of program universe is considered. This observation provides a model for context sensitivity. One result of this feature of program universe is that at any later stage in the evolution of the universe we can always separate any string into two portions, a label string $\mathbf{N}_i(L)$ and a content string $\mathbf{C}_i(S-L)$ and write $P_i(S) = \mathbf{N}_i(L) \|\mathbf{C}_i(S-L)$ with $i \in 1, 2, ..., n_L$, making the context sensitivity explicit. Once we separate labels from content, the permutation invariance we talked about above can only be applied to the columns in the label and/or to the columns in the content parts of the strings separately. Permutations which cross this divide will interfere with any physical interpretation of the formalism we have established up to that point.

In preparation for a more detailed discussion on the foundations of bit-string physics, we note here that the alternatives ADJOIN and TICK correspond *precisely* to the production of a virtual particle represented by a 3-leg "Feynman" diagram, or "3-event", and to the scattering process represented by a 4-leg "Feynman" diagram, or "4-event" respectively. We have to use quotes around

Feynman, because our diagrams obey finite and discrete conservation laws consistent with measurement accuracy. This whole subject will be more fully developed elsewhere, for instance in discussing bit-string scattering theory [75].

Another aspect of program universe is worth mentioning. We note that TICK has a *global* character since a 4-event anywhere in the bit-string universe will necessarily produce a "simultaneous" increase of string length in our space of description. This means that it will be a candidate for representing a coherent cosmological time in an expanding universe model. The time interval to which TICK refers is the shortest meaningful (i.e. finite and discrete) distance that the radius of the universe can advance in our theory divided by the velocity of light. We will return to this idea on another occasion when we discuss cosmology.

3 Lessons from the rejection of the Fine Structure paper

3.1 Background

In preparation for ANPA 9, Christoffer Gefwert[32], David McGoveran[49] and I[61] prepared three papers intended to present a common philosophical and methodological approach to discrete physics. Unfortunately, in order to get the first two papers typed and processed by SLAC, I had to put my name on them, but I want it in the record that my share in Gefwert's and McGoveran's papers amounted mainly to criticism; I made no substantial contribution to their work. We started the report on ANPA 9[60] with these three papers, followed by a paper on Combinatorial Physics by Ted Bastin, John Amson's Parker Rhodes Memorial Lecture (the first in this series), a second paper by John, and a number of first rate contributed papers. Clive Kilmister's concluding remarks closed the volume. I went to considerable trouble to get the whole thing into camera ready format and tried to get the volume into the Springer-Verlag lecture note series, but they were unwilling to accept such a mixed bag. They were interested in the first three papers and were willing to discuss what else to include, but I was unwilling to abandon my comrades at ANPA by dropping any of their contributions to ANPA 9. We ended up publishing the proceedings ourselves, with some much needed help on the physical production from Herb Doughty, which we gratefully acknowledge.

David and I did considerably more work on my paper, and I tried to get it into the mainstream literature, but to no avail. Our joint version ended up in *Physics Essays*[73]. In the interim David had seen how to calculate the fine structure of hydrogen using the discrete and combinatorial approach, and presented a preliminary version at ANPA 10[46]. I was so impressed by this result (see below) that I tried to get it published in *Physical Review Letters*. It was rejected even after we rewrote it in a vain attempt to meet the referee's objections. In order for the reader to form his

own opinion about this rejection, I review the paper[51] here and quote extensively from it.

The first three pages of the paper reviewed the arguments leading to CH and the essential results already achieved. These will already be familiar to the careful reader of the material given above. With this as background we turned to the critical argument:

We consider a system composed of two masses, m_p and m_e — which we claim to have computed from first principles [62] in terms of \hbar , c and $G_{[Newton]}$ — and identified by their labels using our quantum number mapping onto the combinatorial hierarchy [73]. In this framework, their mass ratio (to order α^3 and $(m_e/m_p)^2$) has also been computed using only \hbar , c and 137. However, to put us in a situation more analogous to that of Bohr, we can take m_p and m_e from experiment, and treat 1/137 as a counting number representing the coulomb interaction; we recognize that corrections of the order of the square of this number may become important one we have to include degrees of freedom involving electron-positron pairs. We attribute the binding of m_e to m_p in the hydrogen atom to coulomb events, i.e. only to those events which involve a specific one of the 137 labels at level 3 and hence occur with probability 1/137; the changes due to other events average out (are *indistinguishable* in the absence of additional information). We can have any periodicity of the form 137j where j is any positive integer. So long as this is the only periodicity, we can write this restriction as 137j steps = 1 coulomb event. Since the internal frequency 1/137j is generated independently from the zitterbewegung frequency which specifies the mass scale, the normalization condition combining the two must be in quadrature. We meet the bound state requirement that the energy E be less than the system rest energy $m_{ep}c^2$ (where $m_{ep}=m_em_p/(m_e+m_p)$ is used to take account of 3-momentum conservation) by requiring that $(E/\mu c^2)^2[1+(1/137N_B)^2]=1$. If we take $e^2/\hbar c = 1/137$, this is just the relativistic Bohr formula[14] with N_B the principle quantum number.

[Here I inserted into McGoveran's argument a discussion of the Bohr formula and how it might be derived from dispersion theory. This insertion was motivated by the vain hope that any referee would see that our reasoning was in fact closely related to standard physics. We will look at this result, called the handy-dandy formula, in a new way in the section of this paper carrying that title.]

The Sommerfeld model for the hydrogen atom (and, for superficially different but profoundly similar reasons [12], the Dirac model as well) requires two *independent* periodicities. If we take our reference period j to be integer and the second period s to

differ from an integer by some rational fraction Δ , there will be two minimum values $s_0^{\pm} = 1 \pm \Delta$, and other values of s will differ from one or the other of these values by integers: $s_n = n + s_0$. This means that we can relate ("synchronize") the fundamental period j to this second period in two different ways, namely to

$$137j \frac{steps}{(coulomb\ event)} + 137s_0 \frac{steps}{(coulomb\ event)} = 1 + e = b_+ \tag{7}$$

or to

$$137j \frac{steps}{(coulomb\ event)} - 137s_0 \frac{steps}{(coulomb\ event)} = 1 - e = b_-$$
 (8)

where e is an event probability. Hence we can form

$$a^{2} = j^{2} - s_{0}^{2} = (b_{+}/137)(b_{-}/137) = (1 - e^{2})/137^{2}$$
(9)

Note that if we want a finite numerical value for a, we cannot simply take a square root, but must determine from context which of the symmetric factors [i.e. (1 - e) or (1 + e)] we should take (c.f. the discussion about factoring a quadratic above). With this understood, we write $s_n = n + \sqrt{j^2 - a^2}$.

We must now compute the probability e that j and s are mapped to the same label, using a single basis representation constructed within the combinatorial hierarchy. We can consider the quantity a as an event probability corresponding to an event A generated by a global ordering operator which ultimately generates the entire structure under consideration. Each of the two events j and s can be thought of as derived by sampling from the same population. That population consists of 127 strings defined at level three of the hierarchy. In order that j and s be independent, at least the last of the 127 strings generated in the construction of s (thus completing level three for s) must not coincide with any string generated in the construction of j. There are 127 ways in which this can happen.

There is an additional constraint. Prior to the completion of level three for s, we have available the $m_2 = 16$ possible strings constructed as a level two representation basis to map (i.e. represent) level three. One of these is the null string and cannot be used, so there are 15 possibilities from which the actual construction of the label for s that completes level 3 are drawn. The level can be completed just before or just after some j cycle is completed. So, employing the usual frequency theory of probability, the expectation e that j and s as constructed will be indistinguishable is $e = 1/(30 \times 127)$.

In accordance with the symmetric factors (1 - e) or (1 + e) the value e can either subtract from or add to the probability of a coulomb event. These two cases correspond

to two different combinatorial paths by which the independently generated sequences of events may close (the "relative phase" may be either positive or negative). However we require only the probability that all s_0 events be generated within one period of j, which is 1-e. Hence the difference between j^2 and s^2 is to be computed as the "square" of this "root", $j^2 - s_0^2 = (1 - e)^2$. Thus, for a system dynamically bound by the coulomb interaction with two internal periodicities, as in the Sommerfeld or Dirac models for the hydrogen atom, we conclude that the value of the fine structure constant to be used should be

$$\frac{1}{a} = \frac{137}{1 - \frac{1}{30 \times 127}} = 137.0359 \ 674... \tag{10}$$

in comparison to the accepted empirical value of [1]

$$\frac{1}{\alpha} \simeq 137.0359 \ 895(61) \tag{11}$$

Now that we have the relationship between s, j and a, we consider a quantity H' interpreted as the energy attribute expressed in dynamical variables at the 137j value of the system containing two periods. We represent H' in units of the invariant system energy μc^2 . The independent additional energy due to the shift of s_n relative to j for a period can then be given as a fraction of this energy by $(a/s_n)H'$, and can be added or subtracted, giving us the two factors $(1 - (a/s_n)H')$ and $(1 + (a/s_n)H')$. These are to be multiplied just as we multiplied the factors of a above, giving the (elliptic) equation $(H')^2/(\mu 2c^4) + (a^2/s_n^2)(H')^2/\mu^2c^4 = 1$, Thanks to the previously derived expression of $s = n + s_0$ this can be rearranged to give us the Sommerfeld formula[87]

$$H'/\mu c^2 = \left[1 + \frac{a^2}{(n + \sqrt{j^2 - a^2})^2}\right]^{-1/2}$$
 (12)

Several corrections to our calculated value for α can be anticipated,....

3.2 The rejection

It is obvious to any physicist that if an understandable theory can be constructed which allows one to calculate the fine structure constant and Newton's gravitational constant to high accuracy, it should be possible to create a major paradigm shift in theoretical physics. But even though David McGoveran[46, 51] had showed us how to add four more significant figures to the calculation of the inverse fine structure constant, we were unable to make the chain of thought understandable to most of the physics community. To quote an anonymous referee for *Physical Review Letters*:

I recommend that this letter be rejected. How happy we should all be to publish a physical theory of the fine structure constant! Any such theory, right or wrong, would be worth publishing. But this letter does not contain a theory which might be proved right or wrong. The formula for the fine-structure constant comes out of a verbal discussion which seems to make up its own rules as it goes along. Somewhere underlying the discussion is a random process, but the process is never precisely defined, and its connection to the observed quantities is not explained. I see no way by which the argument of this letter could be proved wrong. Hence I conclude that the argument is not science.

It should be obvious already that, because of my professional background, I have some sympathy with this criticism. In fact, though I was careful not to discuss this with the people in ANPA, for a number of years my research into the meaning of the CH was, in a sense, aimed at giving the canonical ANPA arguments sufficient precision so that they *could* be proved wrong. Then, I could drop my involvement with these ideas and get back to doing more conventional physics. What convinced me that ANPA must be on the right track was, in fact, the McGoveran calculation and his later extension of the same ideas to yield several more mass ratios and coupling constants in better agreement with experiment [47]. Only this past year have we succeeded in getting two publications about discrete physics into leading mainstream physics journals [35, 36]. But the basic canonical calculations are, even today, not in the kind of shape to receive that blessing. This is not the place to review my disheartening attempts to get this and other ANPA calculations before a broader audience. As an illustration of this failed strategy of hoping that the quality of our results would do the job by itself I remind you in the next section what these results were.

4 Quantitative Results up to ANPA 17

We emphasize that the only experimental constants needed as input to obtain these results are \hbar , c and m_p .

The bracketed rational fraction corrections given in bold face type are due to McGoveran[47]. The numerical digits given in bold face type emphasize remaining discrepancies between calculated and observed values.

Newton's gravitational constant (gravitation):

$$G_N^{-1} \frac{\hbar c}{m_p^2} = [2^{127} + 136] \times [\mathbf{1} - \frac{\mathbf{1}}{\mathbf{3} \cdot \mathbf{7} \cdot \mathbf{10}}] = 1.693 \ 31 \dots \times 10^{38}$$

$$experiment = 1.693 \ \mathbf{58}(21) \times 10^{38}$$

The fine structure constant (quantum electrodynamics):

$$\alpha^{-1}(m_e) = 137 \times \left[1 - \frac{1}{30 \times 127}\right]^{-1} = 137.0359 \text{ } 674....$$

$$experiment = 137.0359 \text{ } 895(61)$$

The Fermi constant (weak interactions — β -decay):

$$G_F m_p^2 / \hbar c = [256^2 \sqrt{2}]^{-1} \times [1 - \frac{1}{3 \cdot 7}] = 1.02 \ 758 \dots \times 10^{-5}$$

$$experiment = 1.02 \ 682(2) \times 10^{-5}$$

The weak angle (gives weak electromagnetic unification, the Z_0 and W^{\pm} masses).

$$sin^2 \theta_{Weak} = 0.25 \left[1 - \frac{1}{3 \cdot 7}\right]^2 = 0.2267...$$

 $experiment = 0.2259(46)$

The proton-electron mass ratio (atomic physics):

$$\frac{m_p}{m_e} = \frac{137\pi}{\langle x(1-x) \rangle \langle \frac{1}{y} \rangle} = \frac{137\pi}{(\frac{3}{14})[1+\frac{2}{7}+\frac{4}{49}](\frac{4}{5})} = 1836.15 \ \mathbf{1497} \dots$$

$$experiment = 1836.15 \ 2701(37)$$
(13)

The standard model of quarks and leptons (quantum chromodynamics):

The pion-electron mass ratios

$$m_{\pi}^{\pm}/m_e = 275[\mathbf{1} - \frac{\mathbf{2}}{\mathbf{2} \cdot \mathbf{3} \cdot \mathbf{7} \cdot \mathbf{7}}] = 273.12 \ \mathbf{92} \dots$$

$$experiment = 273.12 \ 67(4)$$

$$m_{\pi^0}/m_e = 274[\mathbf{1} - \frac{\mathbf{3}}{\mathbf{2} \cdot \mathbf{3} \cdot \mathbf{7} \cdot \mathbf{2}}] = 264.2 \ \mathbf{143} \dots$$

 $experiment = 264.1 \ 373(6)]$

The muon-electron mass ratio:

$$m_{\mu}/m_e = 3 \cdot 7 \cdot 10[1 - \frac{3}{3 \cdot 7 \cdot 10}] = 207$$

$$experiment = 206.768 \ 26(13)$$

The pion-nucleon coupling constant:

$$G_{\pi N\bar{N}}^2 = \left[\left(\frac{2M_N}{m_\pi} \right)^2 - 1 \right]^{\frac{1}{2}} = \left[195 \right]^{\frac{1}{2}} = 13.96....$$

experiment = 13.3(3), or greater than 13.9

I eventually came to the conclusion that the only way to get the attention of the establishment would be to show, in detail, that these results can be derived from a finite and discrete version of relativistic quantum mechanics (it turns out, in a finite and discrete Foch space) which is compatible with most of the conventional approach. The rest of the paper is devoted to a sketch of what I think are constructive accomplishments in that direction. The next section is a draft of the start of a paper illustrating the new strategy.

5 The Handy-Dandy Formula

One essential ingredient missing from current elementary particle physics is a non-perturbative connection between masses and coupling constants. We believe that one reason that contemporary conventional approaches to relativistic quantum mechanics fail to produce a simple connection between these two basic classes of parameters is that they start by quantizing classical, manifestly covariant continuous field theories. Then the uncertainty principle necessarily produces infinite energy and momentum at each space-time point. While the renormalization program initiated by Tomonoga, Schwinger, Feynman and Dyson succeeded in taming these infinities, this was only at the cost of relying on an expansion in powers of the coupling constant. Dyson[19] showed in 1952 that this series cannot be uniformly convergent, killing his hope that renormalized QED might prove to be a fundamental theory [85]. Despite the technical and phenomenological successes of non-Abelian gauge theories, this difficulty remains unresolved at a fundamental level. What we propose here as a replacement is an expansion in particle number rather than in coupling constant. The first step in this direction already yields a simple formula with suggestive phenomenological applications, as we now show.

We consider a two-particle system with energies e_a , e_b and masses m_a , m_b which interact via the exchange of a composite state of mass μ . We assume that the exterior scattering state is in a coordinate system in which the particles have momenta of equal magnitude p but opposite direction. The conventional S-Matrix approach starts on energy-shell and on 3-momentum shell with the algebraic connections[6]

$$e_a^2 - m_a^2 = p^2 = e_b^2 - m_b^2$$

$$M^2 = (e_a + e_b)^2 - |\vec{p}_a + \vec{p}_b|^2$$

$$|\vec{p}|(M; m_a, m_b) = \frac{[(M^2 - (m_a + m_b)^2)(M^2 - (m_a - m_b)^2)]^{\frac{1}{2}}}{2M}$$
(14)

but then requires an analytic continuation in M^2 off mass shell. Although this keeps the problem finite in a sense, it leads to a non-linear self-consistency or *bootstrap* problem from which a systematic development of *dynamical* equations has yet to emerge.

We take our clue instead from non-relativistic multi-particle scattering theory [27, 92, 93, 94, 28, 3, 97] in which once a two-particle bound state vertex opens up, at least one of the constituents must interact with a third particle in the system before the bound state can re-form. This eliminates the singular "self energy diagrams" of relativistic quantum field theory from the start. Further, the algebraic structure of the Faddeev equations automatically guarantees the unitarity of the three particle amplitudes calculated from them [31]. The proof only requires the unitarity of the two-body input [54, 57]. This suggests that it might be possible to develop an "on-shell" or "zero range" multi-particle scattering theory starting from some two-particle scattering amplitude formula which guarantees s-wave on-shell unitarity.

In order to implement our idea, rather than use Eq.15 we define the parameter k^2 , which on shell is the momentum of either particle in the zero 3-momentum frame, in terms of the variable s which in the physical region runs from $(m_a + m_b)^2$ (i.e. elastic scattering threshold) to the highest energy we consider by

$$k^{2}(s; m_{a} + m_{b}) = s - (m_{a} + m_{b})^{2}$$
(15)

Then we can insure on-shell unitarity for the scattering amplitude T(s) with the normalization $Im\ T(s) = \sqrt{s - (m_a + m_b)^2} |T|^2$ in the physical region by

$$T(s) = \frac{e^{i\delta(s)}\sin\delta(s)}{\sqrt{s - (m_a + m_b)^2}} = \frac{1}{k \cot \delta(s) - i\sqrt{s - (m_a + m_b)^2}}$$
$$= \frac{1}{\pi} \int_{(m_a + m_b)^2}^{\infty} ds' \frac{\sqrt{s' - (m_a + m_b)^2} |T(s')|^2}{s' - s - i\epsilon} = \frac{2}{\pi} \int_0^{\infty} dk' \frac{\sin^2\delta(k')}{k^2 - (k')^2 - i\epsilon}$$
(16)

We arrived at this way of formulating the two-body input for multi-particle dynamical equations in a rather circuitous way. It turns out that this representation does indeed lead to well defined and soluble zero range three and four particle equations of the Faddeev-Yakubovsky type[57, 75], and that primary singularities corresponding to bound states and CDD poles [17] can be introduced and fitted to low energy two particle parameters without destroying the unitarity of the three and four

particle equations. However, if we adopt the S-Matrix point of view which suggests that elementary particle exchanges should appear in this non-relativistic model as "left hand cuts" starting at $k^2 = -\mu_x^2/4$, where μ_x is the mass of the exchanged quantum [72], then we discovered[57] that the unitarity of the 3-body equations can no longer be maintained; our attempt to use this model as a starting point for doing elementary particle physics was frustrated.

We concluded that a more fundamental approach was required, in the pursuit of which[11, 73] the non-perturbative formula which is the subject of this paper was discovered[51]. However the reasoning was considered so bizarre as, according to one referee, not even to qualify as science. This paper aims to rectify that deficiency by carrying through the derivation in the context of a relativistic scattering theory, which we will call *T-Matrix theory* in order to keep it distinct from the more familiar S-Matrix theory from which it evolved. Thanks to a comment by Castillejo[16] in the context of our treatment of the fine structure of the spectrum of hydrogen[51], we finally realized that the success of our new approach required us from the start to view our *T-Matrix* as embedded in a multi-particle space. This can be accomplished using the relativistic kinematics of Eq.16 rather than of Eq.15 for the off-shell extension which leads to dynamical equations.

As we know from earlier work on partial wave dispersion relations[72], if we know that the scattering amplitude has a pole at $s_{\mu} = \mu^2$, or equivalently at $k^2 + \gamma^2 = 0$ where $\gamma = +\sqrt{(m_a + m_b)^2 - \mu^2}$ then a subtraction in the partial wave dispersion relation given by Eq.17 easily accommodates the constraint while preserving on-shell unitarity in the physical region. This allows us to define the dimensionless coupling constant g^2 as the "residue at the bound state pole" with appropriate normalization. We choose to do this by the alternative definition of T(s) given below:

$$T(s; g^{2}, \mu^{2}) = \frac{g^{2}\mu}{s - \mu^{2}} = \frac{g^{2}\mu}{k^{2}(s) + \gamma^{2}}$$

$$= \frac{1}{k \cot \delta(s) + ik(s)}$$
(17)

Consistency with the dispersion relation, assuming a constant value for g^2 , then requires that at $k^2 = 0$

$$T((m_a + m_b)^2); g^2, \mu^2) = \frac{1}{\gamma} = \frac{g^2 \mu}{\gamma^2}$$

 $k \ ctn \ \delta((m_a + m_b)^2) = \gamma$ (18)

Consequently $g^2\mu = \gamma$ and by taking γ^2 also from Eq. 190 we obtain our desired result, the handy-dandy formula connecting masses and coupling constants:

$$(g^2\mu)^2 = (m_a + m_b)^2 - \mu^2 \tag{19}$$

In the non-relativistic context where $\gamma_{NR}^2 = 2m_{ab}\epsilon_{ab}$, $m_{ab} = m_a m_b/(m_a + m_b)$, $\epsilon_{ab} = m_a + m_b - \mu$, this evaluation of the value of k ctn δ at low energy is equivalent to assuming that the phase shift is given by the mixed effective range expansion[53]:

$$k \operatorname{ctn} \delta = \gamma + k^2/\gamma = -\gamma + (k^2 + \gamma^2)/\gamma \tag{20}$$

corresponding to the zero range bound state wave function $r\psi(r) = e^{-\gamma r}$ which assumes its asymptotic form very close to point where the positions of the two particles coincide. As Weinberg discusses in considerable detail in his papers on the quasi-particle approach [92, 93, 94], this constraint requires the bound state to be purely composite — i.e. to contain precisely two particles with no admixture of effects due to other degrees of freedom. We believe that his analysis supports our contention that we can claim the same interpretation for our relativistic model of a bound state, and hence that we have derived the proper two-particle input for relativistic dynamical n-particle equations of the Faddeev-Yakubovsky type. These equations, which are readily solved for three and four particle systems, will be presented on another occasion [75].

What follows next is an unsystematic presentation of results, some of which were initially obtained using the *combinatorial hierarchy*[11, 73], but which we now claim to have placed firmly within at least the phenomenology of standard elementary particle physics.....

THE TIC-TOC LABORATORY: A Paradigm for Bit-String Physics

Just prior to ANPA 19 I will be attending a conference organized by Professor Zimmermann entitled *NATURA NATURANS: Topoi of Emergence*. The following notes are intended to serve as raw material for my presentation there. Some of these ideas came out of extensive correspondence I have had with Ted and Clive following ANPA 18, and owe much to their comments. In particular the section 6 should be compared to Clive's discussion of a scientific investigation in his paper in these proceedings[39].

I would also like to remind you before we start of Eddington's parable that if we set out to measure the length of the fish in the sea, and we find that they are all greater than one inch long we have the option of concluding (a) that all the fish in the sea are greater than one inch long or (b) that we are using a net with a one inch mesh. Thinking of my approach in this way, I seem to be finding out that because I insist on finite and discrete measurement accuracy together with standard methodological principles. I am bound to end up with something that looks like a finite and discrete relativistic quantum mechanics that has the "universal constants" we observe in the laboratory. Whether the cosmology we observe is also constrained to the same extent is an interesting question. My guess is that we will find that historical contingency plays a significant role.

6 A Model for Scientific Investigation

I restrict our formalism so that it can serve as an abstract model for physical measurement in the following way.

We assume that we encounter entities one at a time, save an entity so encountered, compare it with the next entity encountered, decide whether they are the same or different, and record the result. If they are the same, we record a "0" and if they are different we record a "1". The first of the two entities encountered is then discarded and the second saved, ready to be compared to the next entity encountered. The recursive pursuit of this investigation will clearly produce an ordered string of "0" 's and "1" 's, which we can treat as a bit-string. We further assume that this record — which is our abstract version of a laboratory notebook — can be duplicated, communicated other investigators, treated as the input tape for a Turing machine, cut into segments which can be duplicated, combined and compared using our bit-string operations, the results recorded, and so on.

Our second assumption is that if we cut this tape into segments of length N and determine how many such segments have the Hamming measure a, the probability we will find the integer a, given N will approach $2^{-N} \frac{N!}{a!(N-a)!}$ in the sense of the law of large numbers. Without further tests all such strings characterized by the two integers $a \leq N$ will be called indistinguishable. It should be obvious that I make this postulate in order to be able to, eventually, derive the Planck black body spectrum from my theory. Remember that Planck's formula has stood up to all experimental tests for 97 years, a remarkable achievement in twentieth century physics! We have recently learned that in fact it also represents to remarkable precision the cosmic background radiation at $2.73^{\circ}K$. For those who want to know how and why the quantum revolution started with the discovery of Planck's formula, rather than just myths about what happened, I strongly recommend Kuhn's last major work[41].

Any further structure coming out of our investigation is to be found using the familiar operation of discrimination $\mathbf{a} \oplus \mathbf{b}$ between two strings \mathbf{a} , \mathbf{b} of equal length, by concatenation $\mathbf{a} \parallel \mathbf{b}$ (which doubles the string length for equal length strings), and by taking the Dirac inner product $\mathbf{a} \cdot \mathbf{b}$ which takes two strings out of the category of bit-strings and replaces them by a positive integer. This third operation is also how we determine the Hamming measure of a single string: $\mathbf{a} \cdot \mathbf{a} \equiv a$. It will become our abstract version of quantum measurement, which we interpret as the determination of a cardinal.

Clearly the category change between "bit-string" and "integer" is needed if we are to have a theory of quantitative measurement. I take this to be the hallmark of physics as a science.

The category change produced by taking the inner product allows us to relate two strings which

combine by discrimination to the integer equation:

$$2\mathbf{a} \cdot \mathbf{b} = a + b - (\mathbf{a} \oplus \mathbf{b}) \cdot (\mathbf{a} \oplus \mathbf{b}) \tag{21}$$

If it is taken as axiomatic that (a) we can know the Hamming measure of a bit string and (b) that this implies that we can know the Hamming measure of the discriminant between two bit-strings, then this *basic bit-string theorem* seems very natural.

Once we start combining bit-strings and recording their Hamming measures, and in particular writing down sequential records of these integers, the analysis clearly becomes *context sensitive*. It is our abstract model for a *historical record*.

The underlying philosophy is the assumption that in appropriate units any physical measurement can be abstractly represented by a positive integer with an uncertainty of $\pm \frac{1}{2}$. If we were using real numbers, this would be expressed by saying that the value of the physical quantity represented by $n \pm \frac{1}{2}$ has a 50% chance of lying in the interval between $n - \frac{1}{2}$ and $n + \frac{1}{2}$. But in discrete physics, such a statement is meaningless in the sense used by operationalists. Clearly, part of our conceptual problem is to develop a language describing the uncertainty in the measurement of integers which does not require us to construct the real numbers.

7 Remark on Integers

It is clear from our comment on measurement accuracy that we will find it useful to talk about half-integers as well as integers. This will also be useful when we come to talk about angular momenta and other "non-commuting observables" in our language. But how much farther must we go beyond the positive integers? It was Kronecker who said "God gave us the integers. All else is the work of man." One of our objectives is to keep this extra work to a minimum.

I am certain that the largest string length segment we will need to construct the quantum numbers needed to analyze currently available data about the observable universe of physical cosmology and particle physics is 256, and that all we need do with such segments is to combine or compare or reduce them by the operations listed above, i.e. discrimination, concatenation and inner product. Using as a basis bit-strings of length 16W, I also see how to represent negative integers, positive and negative imaginary integers, and complex integer quaternions. Discussion of how far we need go in that direction, or into using rational fractions other than $\frac{1}{2}$ is, in my opinion, best left until we find a crying need to do so. In any case, we have to lay considerable groundwork before we do.

For the moment we assume all we need know about the integers is that

$$1+0=1=0+1; \quad 1\times 0=0=0\times 1; \quad 1+1=2$$
 (22)

that we can iterate the third equation to obtain the counting numbers up to some largest integer N that we pick in advance as adequate for the purpose at hand, and that given any integer n so generated other than 0 or N, any second integer n' will be greater than, equal to, or less than the first. That is, we assume that the three cases

$$n' > n \quad or \quad n' = n \quad or \quad n' < n \tag{23}$$

are disjoint. This already implies that we can talk about larger integers [34].

I have found that McGoveran's phrase "naming a largest integer in advance", used above, needs be give more structure in my theory. I assume that all the quantum numbers I need consider can be obtained using strings of length 256 or less. If we have 2^{256} such strings, we have more than enough to count— in the sense of Archimedes $Sand\ Reckoner$ — the number electrons and nuclei ("visible matter") in our universe. The mass of the number of nucleons of protonic mass needed to form these nuclei is considerably less than current estimates of the "closure mass" of our universe, leaving plenty of room for the observed "dark matter". I also believe that considerably less than the 256! orders in which we could combine the 2^{256} distinct strings of length 256 will suffice to provide the raw material for a reasonable model of a historical record of both cosmic evolution and terrestrial biological, social and cultural evolution. Such a model can be correct without being complete.

8 From the Tic-Toc Lab to the Digital Lab

Our model for an observatory is a number of *input* devices which, relying on our general model, produce bit-strings of arbitrary length which we can segment, compare, duplicate and operate on using the three operations $\mathbf{a} \oplus \mathbf{b}$, $\mathbf{a} \parallel \mathbf{b}$, and $\mathbf{a} \cdot \mathbf{b}$, and record the results. Our model for a laboratory adds to these observatory facilities, *output* devices which convert bit-strings into signals we can *calibrate* in the laboratory by disconnecting our input devices from the (unknown) signals coming from outside the laboratory and connecting them to our locally constructed output devices. We require that the results correspond to the predictions of the theory which led to their construction. We then take the critical step from being observers to being *participant observers* by connecting our output devices to the outside of the laboratory and seeing if the input signals from the outside into the laboratory change in a correlated way. Just how we construct input and output devices is a matter of *experimental protocol*, which we must test by having other observer-participants construct similar devices and assuring ourselves that they achieve comparable results to our own. *All* of this "background" is presupposed in what I mean by the phrase "the practice of physics" which Gefwert, McGoveran and I employed in our discussion of methodology at ANPA 9[32, 49, 61]. It will be seen

that from the point of view of theoretical physics I am claiming that all of our operations can, in principle, be reduced to bit-string operations looking at input tapes to the laboratory, preparing output tapes connected to the outside world, and comparing the new inputs which result from these outputs. It will sometimes be convenient to refer to these tapes by more familiar names, such as "clocks", "accumulating counters", etc., without going through the detailed translation into laboratory protocol that is required for the actual practice of physics.

The most important device with which to start either an astronomical observatory or a physical laboratory is a reliable clock. For us a *standard clock* will simply consist of an input device which produces a tape with an alternating sequence of "1" 's and "0" 's, which we will also respectively refer to as *tic*'s and *toc*'s. It may be a device we construct ourselves or something that occurs without our intervention other than what is involved in producing the tape. Note that the fact that we have constructed it still leaves the physical clock *outside* our (theoretical) bit-string world; it remains essentially just as mysterious as, for example, the pulsations of a pulsar, as recorded in our observatory.

We have two types of clock with period 2W: a tic-toc clock in which the sequence of 2W alternating symbols starts with a "1", and a toc-tic clock in which the sequence starts with a "0". We will represent the first by a bit-string which we call $\mathbf{L}(W;2W)$ and the second by a bit-string $\mathbf{R}(W;2W)$. The arbitrariness of the designation R or L corresponds the fact that our choice of the symbols on the bit-strings is also arbitrary, reflecting what we called Amson invariance in our discussion of program universe above. Independent of the specific symbolization, these two bit-strings have the following properties in comparison with each other and with the (unique) anti-null string $\mathbf{I}(2W)$ of length 2W (which could be called a "tic-tic clock"):

$$\mathbf{R} \cdot \mathbf{R} = W = \mathbf{L} \cdot \mathbf{L}$$

$$\mathbf{R} \cdot \mathbf{I} = W = \mathbf{L} \cdot \mathbf{I}$$

$$\mathbf{R} \cdot \mathbf{L} = 0$$

$$\mathbf{I} \cdot \mathbf{I} = 2W$$

$$\mathbf{R} \oplus \mathbf{L} = \mathbf{I}$$
(24)

We now have two calibrated clocks one of which we can use to make measurements, and the second to obtain the *redundant* data which is so useful in checking for experimental error — a matter of laboratory protocol which I could expound on at some length, but will refrain from so doing. We now consider an *arbitrary* signal $\mathbf{a}(a; 2W)$, and compare it with the anti-null string. We must have either that (1) $\mathbf{a} \cdot \mathbf{I} < W$ or that (2) $\mathbf{a} \cdot \mathbf{I} = W$ or that (3) $\mathbf{a} \cdot \mathbf{I} > W$. Actually, we need consider only the first two cases, because if we define $\bar{\mathbf{a}}$ by the equation $\bar{\mathbf{a}} \equiv \mathbf{a} \oplus \mathbf{I}$, we can reduce the third case to

the first simply by replacing \mathbf{a} by $\bar{\mathbf{a}}$. If we know the Hamming measure a, for instance by running the string through an accumulating counter which simply records the number of "l" 's in the string, we do not have to make this test because, independent of the order of the bits in the string the definitions of the inner product, the anti-null string \mathbf{I} and the conjugate string $\bar{\mathbf{a}}$ guarantee that

$$\mathbf{a} \cdot \mathbf{a} = a = \mathbf{a} \cdot \mathbf{I}$$

$$\mathbf{\bar{a}} \cdot \mathbf{\bar{a}} = 2W - a = \mathbf{\bar{a}} \cdot \mathbf{I}$$

$$\mathbf{a} \cdot \mathbf{\bar{a}} = 0$$
(25)

Hence an accumulating counter, which throws away most of the information contained in the bitstring \mathbf{a} , still gives us useful structural information if we know the context in which it is employed to produce its single integer result a.

If we compare the bit-string **a** with our standard clock *before* throwing away the string, we get two additional integers, only one of which is independent. Define these by

$$a_R \equiv \mathbf{a} \cdot \mathbf{R}; \quad a_L \equiv \mathbf{a} \cdot \mathbf{L}$$
 (26)

Without actually constructing them, we now know that there exist in our space of length 2W two bit-strings \mathbf{a}_R \mathbf{a}_L with the following properties

$$\mathbf{a}_{R} \oplus \mathbf{a}_{L} = \mathbf{a}$$

$$\mathbf{a}_{R} \cdot \mathbf{a}_{R} = a_{R} = \mathbf{a} \cdot \mathbf{R}$$

$$\mathbf{a}_{L} \cdot \mathbf{a}_{L} = a_{L} = \mathbf{a} \cdot \mathbf{L}$$

$$\mathbf{a}_{R} \cdot \mathbf{L} = 0 = \mathbf{a}_{R} \cdot \mathbf{a}_{L} = 0 = \mathbf{a}_{L} \cdot \mathbf{R}$$

$$a_{R} + a_{L} = a$$

$$(27)$$

Suppose we have a second arbitrary string **b** coming from some independent input device. Clearly we can get some structural information in the same way as before, succinctly summarized by the three integers b, b_R and b_L and the constraint $b = b_L + b_R$. If the two sources are uncorrelated, these amount to a pair of classical measurements, which we can, given enough data of the same type, analyze statistically by the methods developed in classical statistical mechanics. But if the two sources are correlated and we construct the string $\mathbf{a} \oplus \mathbf{b}$ and take its inner product both with itself and with our standard clock before throwing it away, we will have the starting point for a model of quantum measurement. This is a deep subject, on which much light has been shed by Etter's recent papers on Link Theory [22, 23, 24, 25, 26]. We have started to investigate the connection to bit-string physics[70], but have only scratched the surface. We trust that the more systematic

analysis started in this paper will, eventually, help in bringing the two together. Here, we will, instead, show how our tic-toc laboratory can give us useful information about the world in which it is embedded.

9 Finite and Discrete Lorentz Transformations

We now consider a situation in which our laboratory is receiving two independent input signals one of which, for segments of length 2W, repeatedly gives Hamming measure a and the other 2W - a. Because of our experience with the Doppler shift, we leap to the conclusion that our laboratory is situated between two standard clocks similar to our own which are sending output signals to us. We assume that they are at relative rest but that our own lab is moving toward the one for which the recorded Hamming measure is larger than W and away from the second one for which the recorded Hamming measure is smaller than W. We calculate our velocity relative to these two stationary, signalling tic-toc clocks as $v_{lab} = (a - W)/W$ measured relative to the velocity of light. If our lab is in fact a rocket ship and we have any fuel left, we can immediately test this hypothesis by turning on the motors and seeing if, after they have been on long enough to give us a known velocity increment Δv , our velocity measured relative to these external clocks changes to

$$v' = \frac{v + \Delta v}{1 + v\Delta v} \tag{28}$$

If so, we have established our motion relative to a given, external framework. Rather than go on to develop the bit-string version of finite and discrete Lorentz boosts, which is obviously already implicitly available, I defer that development until we have discussed the more general bit-string transformations developed in the section below on commutation relations. For an earlier approach, see [64].

This situation is not so far fetched as might seem at first glance. Basically, this is how the motion of the earth, and of the solar system as a whole, have been determined relative to the $2.73^{\circ}K$ cosmic background radiation in calibrating the COBE satellite measurements that give us such interesting information about the early universe.

To extend this "calibration" of our laboratory relative to the universe to three dimensions, we need only find much simpler pairs of signals than those corresponding to the background radiation, namely pairs, which for the moment we will call U and D which have the properties

$$\mathbf{U} \cdot \mathbf{U} = W = \mathbf{D} \cdot \mathbf{D}$$

$$\mathbf{U} \cdot \mathbf{I} = W = \mathbf{D} \cdot \mathbf{I}$$

$$\mathbf{U} \cdot \mathbf{D} = 0$$
(29)

$$\mathbf{U} \oplus \mathbf{D} = \mathbf{I}$$

These look just like our standard clock, but compared to it we find that

$$\mathbf{U} \cdot \mathbf{R} = W + \Delta = \mathbf{D} \cdot \mathbf{L}$$

$$\mathbf{U} \cdot \mathbf{L} = W - \Delta = \mathbf{D} \cdot \mathbf{R}$$
(30)

where (for \mathbf{U}, \mathbf{D} distinct from \mathbf{R}, \mathbf{L}) we have that $\Delta \in \{1, 2, ..., W-1\}$. These form the starting point for defining directions and finite and discrete rotations. As has been proved by McGoveran[45], using a statistical result obtained by Feller[29], at most three independent sequences which repeatedly have the same number of tic's in synchrony can be expected to produce such recurrences often enough to serve as a "homogeneous and isotropic" basis for describing independent dimensions, showing that our tic-toc lab necessarily resides in a three-dimensional space.

It is well known that finite and discrete rotations of any macroscopic object of sufficient complexity, such as our laboratory, do not commute. It is therefore useful to develop the non-commutative bit-string transformations before we construct the formalism for finite and discrete Lorentz boosts and rotations as a unified theory. Once we have done so, we expect to understand better why finite and discrete commutation relations imply the finite and discrete version of the free space Maxwell[35] and Dirac [36]equations, which we developed in order to answer some of the conceptual questions raised by Dyson's report[20] and analysis[21] of Feynman's 1948 derivation[30] of the Maxwell equations from Newton's second law and the non-relativistic quantum mechanical commutation relations; we intend to extend our analysis to gravitation because we feel that Tanimura's extension of the Feynman derivation in this direction[89] raises more questions than it answers.

10 Commutation Relations

If we consider three bit-strings which discriminate to the null string

$$\mathbf{a} \oplus \mathbf{b} \oplus \mathbf{h}_{ab} = \mathbf{\Phi} \tag{31}$$

they can always be represented by three *orthogonal* (and therefore *discriminately independent*) strings[66, 70]

$$(\mathbf{n}_{a} \oplus \mathbf{n}_{b} \oplus \mathbf{n}_{ab}) \cdot (\mathbf{n}_{a} \oplus \mathbf{n}_{b} \oplus \mathbf{n}_{ab}) = n_{a} + n_{b} + n_{ab}$$

$$\mathbf{n}_{a} \cdot \mathbf{n}_{b} = 0$$

$$\mathbf{n}_{a} \cdot \mathbf{n}_{ab} = 0$$

$$\mathbf{n}_{b} \cdot \mathbf{n}_{ab} = 0$$

$$(32)$$

as follows

$$\mathbf{a} = \mathbf{n}_{a} \oplus \mathbf{n}_{ab} \Rightarrow a = n_{a} + n_{ab}$$

$$\mathbf{b} = \mathbf{n}_{b} \oplus \mathbf{n}_{ab} \Rightarrow b = n_{b} + n_{ab}$$

$$\mathbf{h}_{ab} = \mathbf{n}_{a} \oplus \mathbf{n}_{b} \Rightarrow h_{ab} = n_{a} + n_{b}$$
(33)

It is then easy to see that the Hamming measures a, b, h_{ab} satisfy the triangle inequalities, and hence that this configuration of bit-strings can be interpreted as representing and integer-sided triangle. However, if we are given only the three Hamming measures, and invert Eq.35 to obtain the three numbers n_a, n_b, n_{ab} , we find that

$$n_{ab} = \frac{1}{2}[+a+b-h_{ab}]$$

$$n_{a} = \frac{1}{2}[+a-b+h_{ab}]$$

$$n_{b} = \frac{1}{2}[-a+b+h_{ab}]$$
(34)

Hence, if either one (or three) of the integers a, b, h_{ab} is (are) odd, then n_a, n_b, n_{ab} are half-integers rather than integers, and we cannot represent them by bit-strings. In order to interpret the angles in the triangle as rotations, it is important to start with orthogonal bit-strings rather than strings with arbitrary Hamming measures.

In the argument above, we relied on the theorem that

if
$$\mathbf{n_i} \cdot \mathbf{n}_j = n_i \delta_{ij}$$
 when $i, j \in 1, 2, ..., N$,
then

$$(\Sigma_{\oplus,i=1}^N \mathbf{n}_i) \cdot (\Sigma_{\oplus,i=1}^N \mathbf{n}_i) = \Sigma_{i=1}^N n_i$$
(35)

which is easily proved [70]. Thus in the case of two discriminately independent strings, under the even-odd constraint derived above, we can always construct a representation of them simply by concatenating three strings with Hamming measures n_a , n_b , n_{ab} . This is clear from a second easily proved theorem:

$$(\mathbf{a}\|\mathbf{b}\|\mathbf{c}\|....) \cdot (\mathbf{a}\|\mathbf{b}\|\mathbf{c}....) = a + b + c +$$
(36)

Note that because we are relying on concatenation, in order to represent two discriminately independent strings \mathbf{a} , \mathbf{b} in this way we must go to strings of length $W \geq a + b + h_{ab}$ rather than simply $W \geq a + b$, as one might have guessed simply from knowing the Hamming measures and the Dirac inner product.

If we go to three discriminately independent strings, the situation is considerably more complicated. We now need to know the seven integers $a, b, c, h_{ab}, h_{bc}, h_{ca}, h_{abc}$, invert a 7 × 7 matrix, and put further restrictions on the initial choice in order to avoid quarter-integers as well as

half-integers if we wish to construct an orthogonal representation with strings of minimum length $W \geq a + b + c + h_{ab} + h_{bc} + h_{ca} + h_{abc}$. We have explored this situation to some extent in the references cited, but a systematic treatment using the reference system provided by tic-toc clocks remains to be worked out in detail.

The problem with non-commutation now arises if we try to get away with the scalars a, a_R, Δ, W arrived at in the last section when we ask for a transformation either of the basis $\mathbf{R}, \mathbf{L} \to \mathbf{U}, \mathbf{D}$ or the rotation of the string \mathbf{a} under the constraint $a_R + a_L = a = a_U + a_D$ while keeping the two sets of basis reference strings fixed. This changes $a_R - a_L$ to a different number $a_U - a_D$, or visa versa. If one examines this situation in detail, this is exactly analogous to raising or lowering j_z while keeping j fixed in the ordinary quantum mechanical theory of angular momentum. Consequently, if one wants to discuss a system in which both j and j_z are conserved, one has to make a second rotation restoring j_z to its initial value. It turns out that, representing rotations by \oplus and bit-strings then gives different results depending on whether j_z is first raised and then lowered or visa versa; finite and discrete commutation relations of the standard form result. We will present the details of this analysis on another occasion. In effect what it accomplishes is a mapping of conventional quantum mechanics onto bit-strings in such a way as to get rid of the need for continuum representations (eg. Lie groups) while retaining finite and discrete commutation relations. Then a new look at our recent results on the Maxwell[35] and Dirac[36] equations should become fruitful.

11 Scattering

We ask the reader at this point to refer back to our section on the Handy Dandy Formula when needed. There we saw (Eq. 17) that the unitary scattering amplitude T(s) for systems of angular momentum zero can be computed at a single energy if we know the phase shift $\delta(s)$ at that energy. For the following analysis, it is more convenient to work with the dimensionless amplitude $a(s) \equiv \sqrt{s - (m_a + m_b)^2} T(s)$, which is related to the tangent of the phase shift by

$$a(s) = e^{i\delta(s)} \tan \delta(s) = \frac{\tan \delta(s)}{1 + i \tan \delta(s)} \equiv \frac{t(s)}{1 + it(s)}$$
(37)

In a conventional treatment, given a real interaction potential V(s,s') = V(s',s), T(s) can be obtained by solving the Lippmann Schwinger equation $T(s,s';z) = V(s,s') + \int ds'' T(s,s'';z) R(s'',s';z) T(s'',s';z)$ with a singular resolvent R and taking the limit $T(s) = \lim_{z \to s + i0^+, s' \to s} T(s,s';z)$. Here we replace this integral equation by an algebraic equation for t(s):

$$t(s) = g(s) + g(s)t(s) = \frac{g(s)}{1 - g(s)}$$
(38)

One can think of this equation as a sequence of scatterings each with probability g(s) which is summed by solving the equation. Here g(s) will be our model of a running coupling constant, which we assume known as a function of energy. We see that if $g(s_0) = 0$ there is no scattering at the energy corresponding to s_0 , while if $g(s_0) = +1$, the phase shift is $\frac{\pi}{2}$ at the corresponding energy and $a(s_0) = -i$; otherwise the scattering is finite.

The above remarks apply in the physical region $s > (m_a + m_b)^2$, where in the singular case a phase shift of $\frac{\pi}{2}$ causes the cross section $4\pi sin^2\delta/k^2$ to reach the unitarity limit $4\pi\lambda_0^2$ where $\lambda_0 = \hbar/p_0$ is the de Broglie wavelength at that energy; this is called a resonance and the cross section goes through a maximum value at that energy. If, as in S-matrix theory, we analyticly continue our equation below elastic scattering threshold, the scattering amplitude is real and the singular case corresponds to a bound state pole in which the two particles are replaced by a single coherent particle of mass μ , within which the particles keep on scattering until some third interaction supplies the energy and momentum needed to liberate them. There can also be a singularity corresponding to a repulsion rather than attraction, which is called a "CDD pole" in S-matrix dispersion theory[17]. The corresponding situation in the physical region is a cross section which never reaches the unitarity limit. To cut a long story short, these four cases correspond to the four roots of the quartic equation (Eq. 19) called the handy-dandy formula, which we repeat here, replacing the running coupling constant by its value at the singularity which we call $g_0 = g(s_0)$

$$(g_0)^4 \mu^2 = (m_a + m_b)^2 - \mu^2 \tag{39}$$

Again to cut a long story short, the model for a running coupling constant which Ed Jones and I are exploring[75] is simply

$$g_{m_a,m_b;\mu}(s) = \sqrt{\frac{\pm[(m_a + m_b)^2 - \mu^2](m_a + m_b)^2}{[k^2(s) - (m_a + m_b)^2]s}} g_{m_a m_b;\mu}(0)$$
(40)

The singularity at s = 0 is included only when m_a and m_b have a bound state of zero mass, usually called a quantum.

We have seen that when $m_a = m_e$, $m_b = m_p$ and $\mu = m_H$ the handy-dandy formula gives the relativistic Bohr formula for the hydrogen spectrum. Replacing m_p by m_e in the formula gives the corresponding formula for positronium (i.e the bound state of an electron-positron pair). But for that system, one can think of the photons produced in electron-positron annihilation as bound states of the pair with zero rest mass. This interaction is important in high energy electron-positron scattering, where it is called "Bhabha scattering". Introducing the $s^{-\frac{1}{2}}$ in this way is supposed to insure that our theory gives the correct Feynman diagram (and hence cross section) for this effect, but until we have checked the detailed derivation and predictions I warn the reader to treat this formula (Eq.42) as a guess rather than as a result actually derived from the theory.

In my paper at ANPA WEST 13[70], I started to explore the connections of this type of scattering theory to bit-strings and to Etter's Link Theory by making the hypothesis that

$$tan\delta_{ab} = \frac{\pm (\mathbf{a} \oplus \mathbf{b}) \cdot (\mathbf{a} \oplus \mathbf{b})}{\mathbf{a} \cdot \mathbf{b}}$$
(41)

Unfortunately the details are about a sketchy as presented here, but at least should provide insight into where I am headed.

12 Quantum Gravity

The initial intent of Eddington, and following him of Bastin and Kilmister, was to achieve the reconciliation of quantum mechanics with general relativity. I emphasize here that they were aware of the problem, and thought they had a research program which might solve it, long before the buzzwords "Grand Unified Theory", "Theory of Everything", "Final Theory" or "Ultimate Dynamical Theory" [90, 67] became popular. In a sense they achieved the first major step with the publication of Ted Bastin's paper in 1966[7]. According to John Amson, that paper came about after many attempts to understand Fredrick's breakthrough [79] had led John to the discovery of discriminate closure[2] which gave more mathematical coherence to the scheme, and also convinced Clive and Fredrick that it was time to publish. In the event, the four authors could not agree on a text in time to meet a deadline, and authorized Bastin to go ahead with his version as sole author. This paper really does unify electromagnetism with gravitation (which was also Einstein's long sought and unachieved goal) in the sense that both coupling constants are derived from a common theory. Ted also correctly identified $(256)^4$ with the weak interactions, but missed the $\sqrt{2}$ needed to connect it numerically with the Fermi constant because he was unfamiliar with the difference between 3vertices (Yukawa-type couplings) and 4-vertices (Fermi-type couplings) in the quantum theory of fields. As we all know, this paper was met by resounding silence. I am optimistic, in spite of my past failures, that the bit-string theory now has enough points of contact with more conventional approaches to fundamental physics to get us all into court.

Since the critical problem for many physicists is how we deal with "quantum gravity", I start there. For weak gravitational fields it makes sense to start in a flat space-time[95]. Then it can be shown that spin 2 gravitons of zero mass lead to the Einstein field equations, but as Meisner, Thorne and Wheeler note[52], the "Resulting theory eradicates original flat geometry from all equations, showing it to be unobservable." Consequently they feel that this approach says nothing about "... the greatest single crisis of physics to emerge from these equations: complete gravitational collapse." My qualitative answer is that this crisis arises from using a continuum theory at short

distance where only a quantum theory makes sense. I now try to make the alternative presented here plausible.

My first step is to establish the existence of quantum gravitational effects for neutral particles, namely neutrons. That neutrons are gravitating objects in the classical sense was proved at Brookhaven soon after the physicists there learned how to extract epithermal neutrons from their high flux reactor and send them down an evacuated pipe a quarter of a mile long. The neutrons fell (within experimental error) by just the amount that Galileo would have predicted. That they are quantum mechanical objects was proved by Overhauser [77] by cutting a single silicon crystal 10 centimeters long into three connected planes and using critical reflection, both calculable from measured n-Si cross sections and demonstrable experimentally, to form in effect a two-slit apparatus for neutrons with the positions known to atomic precision over a distance of ten centimeters. Then the shift in the interference pattern between the case when two beams were both horizontal to the case when they were in the vertical plane with one higher than the other for part of its path was proved to be precisely that predicted by non-relativistic quantum mechanics using the Newtonian gravitational potential in the Schroedinger equation. It was this brilliant experiment which convinced me that quantum mechanics is a general theory and not just a peculiarity in the behavior of electrically charged particles at short distance. In my opinion, Overhauser deserves the Nobel prize for this work, which opened up the study of the foundations of quantum mechanics to high precision experimental investigation. In the hands of Rausch [82] and others this technique has led to many tests of the model of the neutron as a quantum mechanical particle acting coherently with a precisely known mass and magnetic dipole.

Having established that neutral particles react gravitationally to the Newtonian gravitational potential $V_N(m_1, m_2; r) = G_N \frac{m_1 m_2}{r}$ as expected, it makes sense to extend our relativistic bit-string model for the Coulomb potential $V_C(m_1, m_2; r) = \frac{Z_1 Z_2 e^2}{r}$ to the gravitational case. Here Z_1, Z_2 are the electric charges expressed in units of the electronic charge e. If we are guided by Bastin's remark quoted above to the effect that the basic quantization is the quantization of mass, the analogy suggests that there is a (currently unknown) unit of mass, which we will call Δm . Then, to complete the analogy with the Coulomb case, we can replace $\alpha_C = e^2/\hbar c \approx 1/137$ with a much smaller constant $\alpha_N = G_N \Delta m^2/\hbar c$. If we also define $N_i = m_i/\Delta m$ for any particle i with gravitational mass m_i , the quantized version of the Coulomb and Newtonian interactions become formally equivalent, differing only by two dimensionless constants and two quantum numbers, independent of the units of charge or mass:

$$V_C(Z_1, Z_2; r) = Z_1 Z_2 \alpha_C \frac{\hbar c}{r}; \quad V_N(N_1, N_2; r) = N_1 N_2 \alpha_N \frac{\hbar c}{r}$$
(42)

We can now apply the Dyson argument to gravitation with more precision. We have seen

that renormalized QED extended to enough precision to generate $N_e=137$ electron-positron pairs, becomes unstable because of (statistically rare) clumping of clusters with enough electrostatic energy to form another pair. We interpret the fact that this disaster does not occur to the formation of a pion with mass $m_{\pi}\approx 2\times 137m_e$. In the neutral particle case, if one assumes CPT invariance, one can still distinguish fermions from anti-fermions by their spin even if they have no other quantum numbers. Hence, independent of whether or how neutral fermions and anti-fermions interact, we can expect gravitational clumping to occur for each type separately. Recall Dyson's remark that the system is dilute enough so that the non-relativistic potential can be used reliably to estimate the interaction energy of the clump. We know experimentally that Δm is much smaller than the electron mass so that the critical radius of the clump is $\hbar/\Delta mc >> \hbar/m_e c$, so Dyson's comment still applies..

In contrast to the electromagnetic case where the cutoff mass-energy of the pion requires us to go outside QED for the physics, in the gravitational case we have a cutoff energy ready to hand, namely the Planck mass $M_{Pk} \equiv [\hbar c/G_N]^{\frac{1}{2}}$. If we assemble a Planck's mass worth of neutral particles of mass Δm at rest within their own Compton wavelength, i.e. $N_G \Delta m = M_{Pk}$, and nothing else intervenes, they will fall together until they are all within a distance of $\hbar/M_{Pk}c$. At that point they will have a gravitostatic energy $N_G G_N \Delta m M_{Pk}/[\hbar/c\Delta M] = M_{Pk}c^2$, which is just sufficient to contain the kinetic energy they acquired in reaching this concentration. Inserting the definition of the Planck Mass into this gravitational energy equation we find that it is algebraically equivalent to the boundary condition with which we started: $N_G \Delta m = M_{Pk}$. They will form a black hole with the Planck radius. We conclude that $N_G = M_{Pk}/\Delta m$ neutral, gravitating objects of mass Δm at rest within their own Compton wavelength will collapse to a black hole with the Planck radius, a quantum version of the disaster that Wheeler is concerned about.

If there are, in fact, neutral fermions with no other properties than their mass, they would form such such black holes and might serve as a model for the dark matter which we know to be at least ten times as prevalent in the universe as ordinary matter. It then becomes a question in big-bang cosmology whether or not they contribute the needed effects to correlate additional observations. We defer that question to another occasion.

If we assemble enough particles of the types we know about to add up to a Planck mass, they can start collapsing and will radiate much of their energy on the way down to higher concentrations. If attractions are balanced by repulsions they could end up close enough together to form a black hole. However, as Hawking showed, small black holes interact with the "vacuum" outside the event horizon and radiate electromagnetically with the consequence that they are "white hot" and soon evaporate; the calculation was extended to rotating, charged black holes by Zurek and Thorne[98].

At the quantum scale a new possibility enters, namely that a quantum number may be possessed by the system which cannot be radiated away by emitting a single particle with that quantum number while conserving energy, momentum and spin. The obvious candidates for such conserved quantum numbers are baryon number, charge and lepton number, suggesting that the lightest baryon (the proton), the lightest charged lepton (the electron) and the lightest neutral lepton (the electron-type neutrino) are gravitationally stabilized black holes with spin $\frac{1}{2}\hbar$. This idea did not make it into the mainstream literature [63]. We use it freely in what follows. The problem then is to explain why $(M_{Pk}/m_p)^2 \approx 2^{127}$, why $m_p/m_e \approx 1836$ and why $m_{\nu_e}/m_e \leq 5 \times 10^{-5} m_e$.

Before we leave gravitation, however, we need to show within bit-string physics that the graviton has spin 2. We know from our discussion of the handy-dandy formula that we can account for spin $\frac{1}{2}$ electrons, positrons and protons and their interactions with the appropriate spin 1 photons. We have shown elsewhere [65] that we can construct the quantum numbers of the standard model of quarks and leptons. In particular, this will include the electron, muon and tau neutrinos and their anti-particles. Extending this approach to a string of length 10 we can have 6 spin $\frac{1}{2}$ fermions and 2 spin 1 bosons. On another occasion I will show how these construct 5 gravitons and 5 anti-gravitons represented in terms of strings of length 10, and go on from that to make a model for dark matter that can be expected to be approximately 12.7 times as prevalent in the universe as electrons and nucleons.

It remains to show that we can meet the three classical tests of general relativity, a problem met on another occasion [71]. Briefly, any relativistic theory gives the solar red shift (Test 1), the factor of 2 compared to special relativity in the bending of light by the sun comes from the spin 1 of the photon (Test 2), and the factor 6 compared to special relativity for the precession of the perihelion of Mercury[13] from the spin 2 of the graviton (Test 3). The calculation by Sommerfeld on which the third argument partly depends comes from simply replacing the factors $N_i = m_i/\Delta m$ in the Coulomb potential by $E_i/\Delta m$, where E_i includes the changing velocity of the orbiting particle in elliptical orbits, and hence is natural in our theory.

13 STRONG, ELECTROMAGNETIC, WEAK, GRAVI-TATIONAL UNIFICATION (SEWGU): A look ahead

I now show, very briefly, how SEWGU might take shape, if all goes well, giving the zeroth approximation to some old results such as $[\alpha_e^{-1}(0)]_0 = 137$, some new results I believe such as $[m_\tau/m_\mu]_0 = 16$, and some guesses such as $[m_t/m_Z]_0 = 2$ which I have little confidence in. The notation makes no distinction between fact and speculation, so *CAVEAT LECTOR*!

Start with strings of length 8 to label the 6 L,R bare states of the three types of neutrinos ν_e , ν_μ , ν_τ and their anti-particles, together with two slots for three generations, $g_1 = (10)$, $g_2 = (11)$, $g_3 = (01)$. To get masses and energies we have to add content strings and do a detailed analysis using bit-string scattering theory. We expect the following results to emerge

$$\left[\frac{m_{\mu}}{m_{e}}\right]_{0} = 210 = \left[\frac{m_{\nu_{\mu}}}{m_{\nu_{e}}}\right]_{0} \tag{43}$$

$$\left[\frac{m_{\tau}}{m_{\mu}}\right]_{0} = 16 = \left[\frac{m_{\nu_{\tau}}}{m_{\nu_{\mu}}}\right]_{0} \tag{44}$$

together with the massless bosons γ_L , γ_R , γ_C (two states of spin 1 photons with the coulomb interaction as a third state) and g_{2L} , g_{1L} , g_0 , g_{1R} , g_{2R} , g_N (five states of the graviton plus the Newtonian interaction).

The electromagnetic coupling at the mass of the Z_0 , $[\alpha_e^{-1}(m_Z)]_0 = 128$.

The masses of the charged and neutral pion compared to the electron:

$$\left[\frac{m_{\pi^{\pm}}}{m_e}\right]_0 = 275 = 2[\alpha_e^{-1}(0)]_0 + 1 \tag{45}$$

$$\left[\frac{m_{\pi^0}}{m_e}\right]_0 = 274 = 2[\alpha_e^{-1}(0)]_0 \tag{46}$$

We can also use label strings of length eight to get (bare) quarks with eight colors (red,orange,yellow,green,blue,purple,black,white) which can form the colorless pion triplet (π^+, π^0, π^-) and the nucleon-antinucleon doublet (n, p, \bar{n}, \bar{p}) . To identify these as first generation hadrons, we need to extend the string length from 8 to 10. This allows us to go on to strings of length 16 and include the neutrinos with the two generation slots accounting for the shared coupling. After a few years of effort, we expect parameters of the full Cabbibo-Kobayashi-Maskawa coupling scheme to emerge. If this fails, we may have to abandon bit-string physics!

Calling the strong coupling constant α_{π} rather than α_{s} to emphasize the conceptual difference, we are confident that at low energy

$$[\alpha_{\pi}(m_{\pi}^2)]_0 = 1 \tag{47}$$

$$[\alpha_{\pi}^{-1}(4m_p^2)]_0 = 7 \tag{48}$$

At high energy we expect to show that

$$\left[\frac{m_Z}{m_p}\right]_0 = 2 \times 7^2 \tag{49}$$

$$\left[\frac{m_t}{m_Z}\right]_0 = 2\tag{50}$$

where m_t is the top quark mass. If this works out we may be able to predict a new coupling between quarks and leptons that goes beyond the standard model which might explain the anomalous results recently obtained at ZEUS and HERA.

I expect to be able to derive the m_p/m_e formula in a way consistent with McGoveran's last paper on that problem[48], but now directly using bit-string dynamics.

I expect to be able to understand the mapping between Foch space labels and bit-string geometry in terms of the Eulerean rectangular block Kilmister told me about with edges of length 44, 117, 240 using strings of length 256 divided into a label of length 16 and content string of length 240.

Finally, I expect to recover the old result

$$\left[\frac{M_{Pk}^2}{m_p^2}\right]_0 = 2^{127} \tag{51}$$

in terms of a running coupling constant for gravitation normalized by $\alpha_G(M_{Pk}^2) = 1$ using bit-string scattering theory.

14 Epilogue

I realize all too well how sketchy these notes are and apologize for that. I hope to get a systematic and reasonably complete outline of the full theory hammered out in a year or two. "But always at my back I hear time's winged chariot hurrying near. And yonder all before us lie deserts of vast eternity.... The grave's a fine and private place, but none I think do there embrace..." theoretical physics! So I decided to rough out this paper to insure that it is part of the Fixed Past before some singular event in the Uncertain Future terminates my activities.

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